Further maths 2 Bi-Week 1 HW

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Due: TBD

Exercises for chapter 19: Hyperbolic functions

1. The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intercept the point where $x = a$.

   (a) Find the exact value of a, giving your answer in logarithmic form.

   (b) Sketch C_1 and C_2 on the same diagram.

   (c) Find the exact vlaue of the length of the arc of C_1 from 0 to $x = a$. (the arc length formula is given as $\int_a^b \sqrt{1 + (y')^2} dx$)

2. (a) Starting from the definitions of tanh and sech in terms of exponential, prove that

$$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta \tag{1}$$

   (b) The variables x and y are such that $\tanh y = \cos(x + \frac{1}{4}\pi)$. By differentiating the equation with respect to x, show that

$$\frac{dy}{dx} = -\csc(x + \frac{1}{4}\pi) \tag{2}$$

3. (a) Sketch the graph of $y = \coth x$ for $x > 0$ and state the equation of the asymptotic.

   (b) Starting from the definition of coth and csch in terms of exponential, prove that

$$\coth^2 x - \operatorname{csch}^2 x = 1. \tag{3}$$

   The curve C has equation $y = \ln(\coth \frac{x}{2})$ for $x > 0$.

   (c) Show that $\frac{dy}{dx} = -\operatorname{csch} x$.

   (d) It is given that the arc length of C from $x = a$ to $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.
   Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a.

4. Solve $17 \sinh x + 16 \cosh x = 8$.

5. Prove the identity $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$.

6. Solve $\tanh^2 x + 5 \operatorname{sech} x - 5 = 0$ in logarithms.