Further Pure Math 2 (FP2): HW 2
Dr.Zhongmin Jin, Biweek1, Week2.

Full name: ______________________
This Homework is worth a total of **66 points**.

1. Write down the eigenvalues of the matrix $\mathbf{A}$, where [10]

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & -16 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix}$$

Find corresponding eigenvectors. [4]
Let n be a positive integer. Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{A}^n = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}.$$

Find $\mathbf{P}^{-1}$ and $\mathbf{A}^n$.[5]
Hence find $\lim_{n\to\infty} (3^{-n}\mathbf{A}^n)$ [1]

2. The vector **e** is an eigenvector of each of the $n \times n$ matrices **A** and **B**, with corresponding eigenvalues λ and μ respectively. Prove that **e** is an eigenvector of the matrix **AB** with eigenvalue $\lambda\mu$. It is given that the matrix **A**, where 12

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 2 \\ -2 & -2 & -2 \\ 1 & 2 & 2 \end{pmatrix}$$

has eigenvectors $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$. Find the corresponding eigenvalues. [2]

Given that 2 is also an eigenvalue of **A**, find a corresponding eigenvector. [2]

The matrix **B**, where

$$\mathbf{B} = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$$

has the same eigenvectors as **A**. Given that $\mathbf{AB} = \mathbf{C}$, find a non-singular matrix **P** and a diagonal matrix D such that

$$\mathbf{P}^{-1}\mathbf{C}^2\mathbf{P} = \mathbf{D}.$$

[8].

3. The matrix $\mathbf{A}$ is given by [9]

$$\mathbf{A} = \begin{pmatrix} 4 & -5 & 3 \\ 3 & -4 & 3 \\ 1 & -1 & 2 \end{pmatrix}$$

Show that $e = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

Find the other two eigenvalues of $\mathbf{A}$. [4]

The matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{pmatrix} -1 & 4 & 0 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{pmatrix}.$$

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{B}$ and deduce an eigenvector of the matrix $\mathbf{AB}$, stating the corresponding eigenvalue. [3]

4. The square matrix $\mathbf{A}$ has an eigenvalue λ with corresponding eigenvector $\mathbf{e}$. The non-singular matrix $\mathbf{M}$ is of the same order as $\mathbf{A}$. Show that $\mathbf{Me}$ is an eigenvector of the matrix $\mathbf{B}$, where $\mathbf{B} = \mathbf{MA}^{-1}$, and that λ is the corresponding eigenvalue. [3] 11

Let

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$

Write down the eigenvalues of $\mathbf{A}$ and obtain corresponding eigenvectors. [4]

Given that

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

find the eigenvalues and corresponding eigenvectors of $\mathbf{B}$. [4]

5. The square matrix $\mathbf{A}$ has λ as an eigenvalue with e as a corresponding eigenvector. Show that e is an eigenvector of A^2 and state the corresponding eigenvalue. [3] 10

Find the eigenvalues of the matrix $\mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

Find the eigenvalues of $\mathbf{B}^4 + 2\mathbf{B}^2 + 3\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix. [3]

6. The matrix A is given by 14

$$A = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}$$

(a) Find the eigenvalues of A. [10]

(b) Use the characteristic equation of A to find A^{-1}. [4]